An Achievable Rate Region for Interference Channels with Common Information

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Abstract—In this paper, we investigate the interference channel with common information, of which the two senders need send not only the private information but also certain common information to their receivers. We study the channel indirectly via a modified channel. An achievable rate region is first obtained for the modified channel via a random coding scheme termed as the “cascaded” superposition coding, and then quickly extended to one for the interference channel with common information.

I. INTRODUCTION

The interference channel (IC) is one of the fundamental building blocks in communication networks, in which the transmissions between each sender and its corresponding receiver (each sender-receiver pair) take place simultaneously and interfere with each other. The information-theoretic study of such a channel was initiated by Shannon in [1], and has been continued by many others (see [2] and references therein). So far, the capacity region of the general IC remains unknown except for some special cases, such as the IC with strong interference (SIC) [3], a class of discrete additive degraded ICs [4], and a class of deterministic ICs [5]. Alternatively, various achievable rate regions served as inner bounds of the capacity region have been derived for the general IC [6]–[8]. Notably, Carleial [6] obtained an achievable rate region of the discrete memoryless IC by employing a limited form of the general superposition coding scheme [9], successive encoding and decoding. Subsequently, Han and Kobayashi [7] established the best achievable rate region known till date by applying the simultaneous superposition coding scheme consisting of simultaneous encoding and decoding. Most recently, Chong et al. obtained an achievable rate region identical to the Han-Kobayashi region but with a much simplified description, by using a hybrid of Carleial’s successive encoding and the simultaneous decoding.

Most of the prior work on the ICs assumes the statistical independence of the source messages to be transmitted by the senders (see [2] and references therein). However, this assumption becomes invalid in an IC where the senders need transmit not only the private information but also certain common information to their corresponding receivers. Such a scenario is generally modelled as the IC with common information (ICC). Maric et al. [10] derived the capacity region of a special case of the ICC, the strong interference channel with common information (SICC), and their result subsumes the capacity region of the SIC (without common information) [3] as a special case. Moreover, the capacity region can be interpreted as the intersection of the capacity regions of the two associated multiple access channel with common information (MACs) [11], [12].

In this paper, we propose a generalized version of the successive superposition encoding, namely cascaded superposition encoding, which reduces to Carleial’s successive encoding in the absence of common information. With this encoding scheme, the senders’ common information is conveyed through the channel in a cooperative manner. Applying the proposed cascaded encoding scheme along with the simultaneous decoding scheme [7], [8], we derive an achievable rate region for the general two-user discrete memoryless ICC.

II. CHANNEL MODELS AND PRELIMINARIES

A. Discrete Memoryless Interference Channel with Common Information

A discrete memoryless IC is usually defined by a quintuple \((X_t, X_2, Y_1, Y_2)\), where \(X_t = \{x_{1t}, \ldots, x_{nt}\}\) and \(Y_t = \{y_{1t}, \ldots, y_{nt}\}\) for \(t = 1, 2\). The marginal distributions of \(y_1\) and \(y_2\) are given by

\[
p_1(y_1|x_1, x_2) = \sum_{y_2 \in Y_2} p(y_1, y_2 | x_1, x_2),
\]

\[
p_2(y_2|x_1, x_2) = \sum_{y_1 \in Y_1} p(y_1, y_2 | x_1, x_2).
\]

Building upon an IC, we depict an ICC in Fig. 1. Sender \(t\), \(t = 1, 2\), is to send a private message \(w_t \in M_t = \{1, \ldots, M_t\}\) together with a common message \(w_0 \in M_0 = \{1, \ldots, M_0\}\) to its pairing receiver. All the three messages are assumed to be independently and uniformly generated over their respective ranges.

Let \(C\) denote the discrete memoryless ICC defined above. An \((M_0, M_1, M_2, n, P_c)\) code exists for the channel \(C\), if and
only if there exist two encoding functions
\[ f_1 : \mathcal{M}_0 \times \mathcal{M}_1 \to \mathcal{X}_1^n, \quad f_2 : \mathcal{M}_0 \times \mathcal{M}_2 \to \mathcal{X}_2^n, \]
and two decoding functions
\[ g_1 : \mathcal{Y}_1^n \to \mathcal{M}_0 \times \mathcal{M}_1, \quad g_2 : \mathcal{Y}_2^n \to \mathcal{M}_0 \times \mathcal{M}_2, \]
such that \( \max \{P_{e,1}^{(n)}, P_{e,2}^{(n)}\} \leq P_e \), where \( P_{e,t}^{(n)}, t = 1, 2, \)
denotes the average decoding error probability of decoder \( t \), and
is computed by one of the following:
\[ P_{e,1}^{(n)} = \frac{1}{M} \sum_{w_0w_1w_2} p((\hat{w}_0, \hat{w}_1) \neq (w_0, w_1)|(w_0, w_1, w_2)), \]
\[ P_{e,2}^{(n)} = \frac{1}{M} \sum_{w_0w_1w_2} p((\hat{w}_0, \hat{w}_2) \neq (w_0, w_2)|(w_0, w_1, w_2)), \]
where \( M \triangleq M_0M_1M_2 \).

A non-negative rate triple \((R_0, R_1, R_2)\) is achievable for the
cchannel if for any given \( 0 < P_e < 1 \), and for any sufficiently
large \( n \), there exists a \((2^{nR_0}, 2^{nR_1}, 2^{nR_2}, n, P_e)\) code.

The capacity region for the channel \( C \) is defined as the
closure of the set of all the achievable rate triples, while an
achievable rate region for the channel \( C \) is a subset of the
capacity region.

B. Modified Interference Channel with Common Information

To derive an achievable rate region for the ICC, we first
need to be clear about the structure of the information flow
through it. However, this can not be viewed from the original
ICC model clearly, and thus it is difficult for one to carry
out the corresponding information-theoretic analysis. To avoid
such difficulty, we introduce the modified ICC by following
the same approach used in [7].

The modified ICC inherits the same channel characteristics
from its associated ICC, but there are five streams of messages
to be conveyed through the modified channel instead of three
through the associated ICC. The five streams of messages
\( n_0, n_1, l_1, n_2 \) and \( l_2 \) are assumed to be independently and
uniformly generated over the finite sets \( N_0 = \{1, \ldots, N_0\}, \)
\( N_1 = \{1, \ldots, N_1\}, L_1 = \{1, \ldots, L_1\}, N_2 = \{1, \ldots, N_2\}, \) and
\( L_2 = \{1, \ldots, L_2\}, \) respectively.

Denote the modified ICC shown in Fig. 2 by the channel
\( C_m \). An \((N_0, N_1, L_1, N_2, L_2, n, P_e)\) code exists for \( C_m \) if and
only if there exist two encoding functions
\[ f_1 : N_0 \times N_1 \times L_1 \to \mathcal{X}_1^n, \quad f_2 : N_0 \times N_2 \times L_2 \to \mathcal{X}_2^n, \]
and two decoding functions
\[ g_1 : \mathcal{Y}_1^n \to N_0 \times N_1 \times L_1, \quad g_2 : \mathcal{Y}_2^n \to N_0 \times N_2 \times L_2, \]
such that \( \max \{P_{e,1}^{(n)}, P_{e,2}^{(n)}\} \leq P_e \), where the average probabilities
of decoding error denoted by \( P_{e,1}^{(n)} \) and \( P_{e,2}^{(n)} \) are computed as
\[ P_{e,1}^{(n)} = \frac{1}{N_0n_1L_1} \sum_{n_0n_1n_2l_2} p((\hat{n}_0, \hat{n}_1, \hat{l}_1) \neq (n_0, n_1, l_1)), \]
\[ P_{e,2}^{(n)} = \frac{1}{N_0n_1L_2} \sum_{n_0n_1n_2l_2} p((\hat{n}_0, \hat{n}_2, \hat{l}_2) \neq (n_0, n_2, l_2)), \]
where \( N \triangleq N_0N_1L_1N_2L_2 \).

A non-negative rate quintuple \((R_0, R_{12}, R_{11}, R_{21}, R_{22})\) is
achievable for the channel \( C_m \) if for any given \( 0 < P_e < 1 \)
and sufficiently large \( n \), there exists a \((2^{nR_0}, 2^{nR_{12}}, 2^{nR_{11}}, 2^{nR_{21}}, 2^{nR_{22}}, n, P_e)\) code for \( C_m \).

Remark 1: It should be noted that compared with Fig. 2 in [7],
our modified channel depicted in Fig. 2 does not include
the index \( \hat{n}_2 \) (or \( \hat{n}_1 \)) in the decoded message vector at decoder
1 (or decoder 2). This is due to the observation made in [8]
that, although receiver 1 (or receiver 2) attempts to decode
the crossly observable private message \( n_2 \) (or \( n_1 \)), it is not
essential to include decoding errors of such information in
calculating probability of error at the respective receiver.
This is also the reason why we call the two associated channels
of an ICC as MACC-like channels instead of MACCs.

Lemma 1: If \((R_0, R_{12}, R_{11}, R_{21}, R_{22})\) is achievable for the
channel \( C_m \), then \((R_0 + R_{12} + R_{11}, R_{21} + R_{22})\) is achievable
for the associated ICC \( C \).

Remark 2: Note that with the aid of Lemma 1, an achievable
rate region for the modified ICC can be easily extended
to one for the associated ICC.

III. GENERAL DISCRETE MEMORYLESS INTERFERENCE
CHANNEL WITH COMMON INFORMATION

A. An Achievable Rate Region for the General Discrete Memoryless
ICC

We first introduce three auxiliary random variables \( U_0, U_1 \)
and \( U_2 \) that are defined over arbitrary finite sets \( U_0, U_1, \) and
Let $R_m(p)$ denote the set of all non-negative rate quintuples $(R_0, R_{12}, R_{11}, R_{21}, R_{22})$ such that

\begin{align*}
R_{11} &\leq I(X_1;Y_1|U_0U_1U_2), \\
R_{12} + R_{11} &\leq I(U_1X_1;Y_1|U_0U_2), \\
R_{11} + R_{21} &\leq I(X_1U_2;Y_1|U_0U_1), \\
R_{12} + R_{11} + R_{21} &\leq I(U_1X_1U_2;Y_1|U_0), \\
R_0 + R_{12} + R_{11} + R_{21} &\leq I(U_0U_1X_1U_2;Y_1), \\
R_{22} &\leq I(X_2;Y_2|U_0U_2U_1), \\
R_{21} + R_{22} &\leq I(U_2X_2;Y_2|U_0U_1), \\
R_{22} + R_{12} &\leq I(X_2U_1;Y_2|U_0U_1), \\
R_{21} + R_{22} + R_{12} &\leq I(U_2X_2U_1;Y_2|U_0), \\
R_0 + R_{21} + R_{22} + R_{12} &\leq I(U_0U_2X_2U_1;Y_2), 
\end{align*}

for some fixed joint probability distribution $p(\cdot) \in \mathcal{P}^*$. Note that each of the mutual information terms is computed with respect to the given fixed joint distribution.

Theorem 1: Any element $(R_0, R_{12}, R_{11}, R_{21}, R_{22}) \in R_m(p)$ is achievable for the modified ICC $C_m$ for a fixed joint probability distribution $p(\cdot) \in \mathcal{P}^*$.

Remark 3: The lengthy proof is relegated to the last subsection of this section. Theorem 1 lays a foundation for us to establish an achievable rate region for the general ICC. One can interpret this achievable rate region as an intersection between the achievable rate regions of the two associated MACC-like channels, i.e., inequalities (2)–(6) depict the achievable rate region for one MACC-like channel, and inequalities (7)–(11) depict the other.

Theorem 2: The rate region $R_m$ is achievable for the channel $C_m$ with $R_m = \bigcup_{p(\cdot) \in \mathcal{P}^*} R_m(p)$.\n
Remark 4: Theorem 2 is a direct extension of Theorem 1, and the proof is straightforward and omitted. Note that the rate region $R_m$ is convex, and therefore no convex hull operation or time sharing is necessary. The proof of the convexity is given in the appendix of [13].

Let us fix a joint distribution $p(\cdot) \in \mathcal{P}^*$, and denote by $R(p)$ the set of all the non-negative rate triples $(R_0, R_{12}, R_{11})$ such that $R_1 = R_{12} + R_{11}$ and $R_2 = R_{21} + R_{22}$ for some $(R_0, R_{12}, R_{11}, R_{21}, R_{22}) \in R_m(p)$.

Theorem 3: $R$ is an achievable rate region for the channel $C$ with $R = \bigcup_{p(\cdot) \in \mathcal{P}^*} R(p)$.

Proof: It suffices to prove that $R(p)$ is an achievable rate region for $C$ for any fixed joint probability distribution $p(\cdot) \in \mathcal{P}^*$, while the achievability of any rate triple $(R_0, R_{12}, R_{21}) \in R(p)$ follows immediately from Lemma 1 and Theorem 1.

Remark 5: The main idea, as mentioned before, is that we allow the common information (of rate $R_0$) to be cooperatively transmitted by the two senders, on top of which we treat the private information at each sender as two parts. One part (of rate $R_{12}$ or $R_{21}$) of the private information at each sender is crossly observable to the non-pairing receiver, but not the other part (of rate $R_{11}$ or $R_{22}$). However, as discussed earlier, for each of the two receivers, the crossly observed information is not required to be decoded correctly. This will be elaborated more clearly in the proof of Theorem 1.

Remark 6: One can observe that the rate of the common information, $R_0$, is bounded by only one inequality at each decoder. This is similar to the case of MACC [11], [12], where the rate of the common information is bounded by only one inequality as well. This is due to the perfect cooperation of the two senders in transmitting the common information, and the simultaneous decoding. Details will also be illustrated in the proof of Theorem 1.

Remark 7: Note that the region $R$ is also convex, and one can readily prove it by following procedures in the proof of the convexity of $R_m$ [13].

B. An Explicit Description of the Achievable Rate Region

In order to reveal the geometric shape of the region $R$ depicted in Theorem 3, we derive an explicit description of the region by applying Fourier-Motzkin eliminations [2], [7], [8].

Theorem 4: The rate region $R$ is achievable for the channel $C$ with $R = \bigcup_{p(\cdot) \in \mathcal{P}^*} R(p)$, where $R(p)$ denotes the set of all rate triples $(R_0, R_{12}, R_{21})$ such that

\begin{align*}
R_0 &\leq I(U_0U_1X_1U_2;Y_1), \\
R_0 &\leq I(U_0U_2X_2U_1;Y_2), \\
R_1 &\leq I(U_1X_1Y_1|U_0U_2), \\
R_2 &\leq I(U_2X_2Y_2|U_0U_1), \\
R_1 + R_2 &\leq I(X_1U_2;Y_1|U_0U_1) + I(X_2U_1;Y_2|U_0U_2); \\
R_0 + R_1 + R_2 &\leq I(U_0U_1X_1U_2;Y_1|U_2) + I(X_1Y_1U_0U_2); \\
R_0 + R_1 + R_2 &\leq I(X_1X_2U_2;Y_2|U_0U_1) + I(U_0U_2X_2U_1;Y_2); \\
R_0 + R_1 + 2R_2 &\leq I(U_1X_1U_2;Y_1|U_0) + I(X_1Y_1U_0U_2) + I(X_2U_2Y_2|U_0U_2); \\
R_0 + 2R_1 + R_2 &\leq I(U_0X_1U_2;Y_1) + I(X_1Y_1U_0U_2); \\
R_1 + 2R_2 &\leq I(U_1X_1Y_1U_0U_2); \\
R_0 + R_1 + 2R_2 &\leq I(U_0U_2X_2U_1;Y_2) + I(X_2Y_2|U_0U_1); \\
R_0 + R_1 + 2R_2 &\leq I(U_0U_1X_1U_2;Y_1) + I(X_1Y_1U_0U_2) + I(X_2U_2Y_2|U_0U_2); \\
R_0 + 2R_1 + R_2 &\leq I(U_0X_1U_2;Y_1) + I(X_1Y_1U_0U_2) + I(X_2U_2Y_2|U_0U_2); \\
\end{align*}

for some fixed joint distribution $p(\cdot) \in \mathcal{P}^*$.\n
Remark 8: The close relation between the explicit Chong-Motani-Garg region and the capacity region of a class of deterministic ICs given in [5] was pointed out in [2]. Similarly, we have disclosed that the explicit region for the ICC is also closely related to the capacity region of a class of deterministic interference channels with common information (DICCs) in [13].
C. The Proof of Theorem 1

In this section, we will prove Theorem 1 that is the core of this paper up to here. The general idea is to apply the cascaded superposition encoding and simultaneous decoding. As the following lemma will be frequently used, we state it here before the proof of Theorem 1.

Lemma 2 ([14, Theorem 14.2.3]): Let \( A_1^{(n)} \) denote the typical set for the probability distribution \( p(s_1, s_2, s_3) \), then

\[
P(S_1 = s_1, S_2 = s_2, S_3 = s_3) = \prod_{i=1}^{n} p(s_{1i} | s_{3i}) p(s_{2i} | s_{3i}) p(s_{3i}),
\]

(12)

and transmitted. Since the messages are uniformly generated over their respective ranges, the average error probability over all the possible messages is equal to the probability of error incurred when any message vector is encoded and transmitted. We hence only analyze the probability of decoding error for decoder 1 in details, since the same analysis can be carried out for decoder 2. Without loss of generality, we assume that a source message vector \((n_0, n_1, l_1, n_2, l_2) = (1, 1, 1, 1, 1)\) is encoded and transmitted for the subsequent analysis. We first define the event

\[ E_{ijkl} \triangleq \left\{ \left( U_0(i), U_1(i, j), X_1(i, j, k), U_2(i, l), Y_1 \right) \in A_1^{(n)} \right\}. \]

The possible error events can be grouped into two classes: 1) the codewords transmitted are not jointly typical, i.e., \( E_{1111}^C \) happens; 2) there exist some \((i, j, k, l) \neq (1, 1, 1, 1)\) such that \( E_{ijkl} \) happens \((l \neq 1)\). Thus the probability of decoding error at decoder 1 can be expressed as

\[ P_{e,1} = P(E_{1111}^C \cup \bigcup_{(i,j,k) \neq (1,1,1)} E_{ijkl}). \]

(14)

By applying the union bound, we can upper-bound (14) as

\[
P_{e,1}^C \leq P(E_{1111}^C) + P(\bigcup_{(i,j,k) \neq (1,1,1)} E_{ijkl}) \\
\leq P(E_{1111}^C) + \sum_{i \neq 1} P(E_{1i1}) + \sum_{j \neq 1} P(E_{i1j}) + \sum_{k \neq 1} P(E_{i1k}) \\
+ \sum_{i \neq 1, j \neq 1} P(E_{ij1}) + \sum_{i \neq 1, k \neq 1} P(E_{i1k}) \\
+ \sum_{j \neq 1, k \neq 1} P(E_{ij1}) + \sum_{j \neq 1, l \neq 1} P(E_{ijl}) + \sum_{k \neq 1, l \neq 1} P(E_{ikl}) \]

(15)

Due to the asymptotic equipartition property (AEP) [14], \( P(E_{1111}^C) \) in (15) can be made arbitrarily small as long as \( n \) is sufficiently large. The rest of the fourteen probability terms in (15) can be evaluated through one standard procedure, which is demonstrated as follows. To evaluate \( P(E_{1111}) \), we apply Lemma 2 by letting \( S_1' = (U_0(i), U_1(i, 1), X_1(i, 1, k), U_2(1, l), Y_1) \), \( S_2' = Y_1 \), and \( S_3' = \emptyset \), where \( \emptyset \) denotes the empty set. Note that the assumption of the lemma on the joint distribution of \( S_1', S_2', S_3' \) is satisfied, and thus it follows that

\[ P(E_{1111}) \leq 2^{-n[I(U_0U_1X_1U_2Y_1) - 6\epsilon]}. \]

(16)

Since the case with \( S_3' = \emptyset \) seems not archetypal, we evaluate one more probability term, \( P(E_{1jk1}) \). Again, we use Lemma 2 by letting \( S_1' = (U_1(1, j), X_1(1, j, k)), S_2' = Y_1 \), and \( S_3' = (U_0(1), U_2(1, l)) \) to obtain

\[ P(E_{1jk1}) \leq 2^{-n[I(U_1X_1U_0U_2Y_1) - 6\epsilon]}. \]

(17)
By repeatedly applying Lemma 2, we obtain upper-bounds of the remaining twelve probability terms. Further, we employ these bounds to derive an upper-bound of the probability of decoding error at decoder 1 as

\[
P_{e,1}^{(n)} \leq \epsilon + 2^{nR_0}2^{-n(I(U_0U_1X_1X_2Y_1) - 6\epsilon)} + 2^{nR_{12}}2^{-n(I(U_0U_1X_1Y_1U_2) - 6\epsilon)} + 2^{nR_{11}+R_{21}}2^{-n(I(U_1X_1X_2Y_1U_0) - 6\epsilon)} + 2^{nR_{11}+R_{21}}2^{-n(I(U_1X_1Y_1U_0U_1) - 6\epsilon)} + 2^{nR_{0}+R_{12}+R_{21}}2^{-n(I(U_0U_1X_1X_2Y_1) - 6\epsilon)} + 2^{nR_{0}+R_{13}}2^{-n(I(U_0U_1X_1Y_1U_2) - 6\epsilon)} + 2^{nR_{12}+R_{13}+R_{21}}2^{-n(I(U_1X_1X_2Y_1U_0) - 6\epsilon)} + 2^{nR_{12}+R_{13}+R_{21}}2^{-n(I(U_1X_1Y_1U_0U_1) - 6\epsilon)} + 2^{nR_{0}+R_{12}+R_{13}+R_{21}}2^{-n(I(U_0U_1X_1X_2Y_1) - 6\epsilon)} + 2^{nR_{0}+R_{12}+R_{13}+R_{21}}2^{-n(I(U_0U_1X_1Y_1U_2) - 6\epsilon)} + 2^{nR_{12}+R_{13}+R_{21}}2^{-n(I(U_1X_1X_2Y_1U_0) - 6\epsilon)} + 2^{nR_{12}+R_{13}+R_{21}}2^{-n(I(U_1X_1Y_1U_0U_1) - 6\epsilon)}.
\]

(18)

It is now easy to check that when inequalities (2)–(6) hold and \( n \) is sufficiently large, we have

\[
P_{e,1}^{(n)} \leq 15\epsilon.
\]

(19)

By symmetry, the decoding error probability becomes \( P_{e,2}^{(n)} \leq 15\epsilon \) for decoder 2, when inequalities (7)–(11) hold and \( n \) is sufficiently large. It follows that \( \max\{P_{e,1}^{(n)}, P_{e,2}^{(n)}\} \leq 15\epsilon \), and thus any rate quintuple \((R_0, R_{12}, R_{11}, R_{21}, R_{22}) \in \mathcal{R}_m(p)\) is achievable for the modified ICC \( C_m \) for a fixed joint distribution \( p(x) \in \mathcal{P}^* \).

Remark 9: In what follows, we list a few remarks on the encoding and decoding scheme used in our derivation.

1) We term the above coding scheme the “cascaded superposition coding,” because there are three layers of code with the bottom one \( u_0(i) \) carrying the common information. The second layer consists of \( u_1(i,j) \) and \( u_2(i,l) \). This layer superimposes the part of each sender’s private information, which is crossly observable to the non-pairing receiver, on the bottom layer; while \( x_1(i,j,k) \) and \( x_2(i,l,m) \) form the top layer, and they are generated by superimposing the part of private information which is not crossly observable on top of both the second layer and the bottom layer.

2) The encoding scheme is auxiliary random variable efficient in the sense that it only requires three auxiliary random variables instead of five required if one follows [7] to apply the simultaneous superposition coding scheme. It not only greatly simplifies the description of the achievable rate region in terms of the number of inequalities required, but also has implications on practical code design or implementation of the system in the sense that the number of different codes required is reduced.

3) For the decoding, the simultaneous joint typicality of three layers of codes is examined. It is the reason why we could have to use fourteen inequalities due to (18), but we in fact only use five inequalities (inequalities (2)–(6)) instead. Due to the cascaded superpositioning and simultaneous decoding, \( R_0 \) is only bounded together with other rates by (6) or (11) for each decoder. The advantage of the simultaneous decoding over the successive decoding is also demonstrated with an example of MACC in [15].

IV. CONCLUSIONS

In this paper, we have proposed a random coding scheme for the general two-user discrete memoryless interference channel with common information, and obtained an achievable rate region for it. The derived achievable rate region in fact subsumes the Chong-Motani-Garg region for the general IC as well as the capacity region for the SICC as special cases. More complete results and related proofs are presented in the full version of this work [13].

REFERENCES